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ABSTRACT

Undergraduate mathematics courses in Australia, as elsewhere, have for some time been integrating software into their teaching programs. This international trend is stimulated by the increase in technological resources in general and the impact of symbolic manipulator packages. It has been recommended that mathematics departments re-design courses to make the best use of the increased computer power becoming available. This paper examines some of the issues that are emerging as this process unfolds. It focuses on the computer-based undergraduate courses' attitudes towards mathematics and technology. Studies designing attitude scales for use in programs with computer technology, classifying the range of student-generated questions that emerge when learning of mathematical content interacts with a symbolic manipulator environment, and identifying structural properties associated with the Maple environment that can be identified as linking task demand and student success are discussed. (Contains 24 references.) (ASK)

Digging Beneath the Surface: When Manipulators, Mathematics, and Students Mix

by

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Digging beneath the surface: when Manipulators, Mathematics, and Students mix

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Background

Undergraduate mathematics courses in Australia, as elsewhere, have for some time been integrating software into their teaching programs (Pemberton, 1997), an international trend stimulated by the increase in technological resources in general, and the impact of symbolic manipulator packages, such as *Derive, Mathematica, and Maple* in particular. An Australian report, *Mathematical Sciences-Adding to Australia* (NBEET, 1996) noted then that the mathematical sciences were becoming increasingly laboratory based, with significant implications for how they will be taught. It recommended that mathematics departments re-design courses to make best use of the increased computer power becoming available. This paper looks at some of the issues that are emerging as this process unfolds.

Computer-Based Undergraduate Programs

The form of computer-based instruction varies widely. Olsen (1999) describes one of the most extensive examples of technology used to provide automated instruction. She describes how politicians visiting Virginia Tech's Mathematics Emporium, a 58 000 square foot (1.5-acre) computer classroom:

> see a model of institutional productivity; a vision of the future in which machines handle many kinds of undergraduate teaching duties-and universities pay fewer professors to lecture

On weekdays from 9 am to midnight dozens of tutors and helpers stroll along the hexagonal pods on which the computers are located. They are trying to spot the students who are stuck on a problem and need help. This program appears to be openly driven by economic rationalism. At the other extreme Shneiderman et al (1998) describe a model, in which electronic classroom infrastructure is extensive and expensive, containing full computer and multi-media facilities as well as designer courseware. Courses are scheduled into the electronic classrooms on a semester basis, and must go through a proposal process to make sure that the resources will be used as designed. It is required that full use be made of the interactive, collaborative, multi-media environment.

In between such extremes occur a variety of models of instruction, whose users are concerned in varying degrees about factory production on the one hand, and student understanding on the other. Of those valuing the latter Alavi (1994) imported constructivist principles into computer-based learning by emphasizing that learning is best accomplished by acquiring, generating, analysing, manipulating and structuring information. However Templer et al (1998) raised problems accompanying such efforts to provide meaningful learning that were perceived to arise as a direct result of a symbolic manipulator (*Mathematica*) environment. They noted that typically having mastered the rudiments; the majority of students "began to hurtle through the work, hell bent on finishing everything in the shortest possible time." The following comment, or a close relative, was noted as occurring frequently "I just don't understand what I'm learning here. I mean all I have to do is ask the machine to solve the problem and it's done. What have I learned?"

Kent and Stevenson (1998) drew attention to the potential influence of a decline in the mathematical preparedness of science and engineering undergraduates in some settings. In suggesting that a response has been to simply reduce mathematical content, and to rely on computer based tools to do much of the computation, they question whether mathematical procedures can be learned effectively without appreciation of their place in the structure of mathematics. Their evidence and observation suggested that unless some kind of breakdown in the functionality of some concept or procedure (say integration) was provoked, the student would not focus on the essential aspects of that concept or procedure. On the other hand it was observed that the demands for formal precision that a programming environment places on its user, serves both to expose any fragility in understanding, and to support the building and conjecturing required in the re (construction) of concepts by learners. Templer et al (1998) further commented that an anticipated worry about the non-standard nature of *Mathematica* syntax did not impact severely, and that in fact the symbolic manipulator environment was helpful to the extent that its language was sufficiently close to that of mathematics for the two to be treated in tandem. Similar claims could be made on behalf of *Maple* and *Derive*. Ramsden (1997) however indicated that syntax issues are a concern in other respects. In noting that students were being asked to grapple with some quite complicated, difficult, and potentially irksome syntax he identified a dilemma created by the writing of special purpose modules that provided students with manipulative convenience through exercise of the select button. In avoiding the need to consider other inputs such as occurs when they must be typed individually, he saw an increase in mystery, and the development of a 'black box' syndrome. Also with respect to the distribution of activity Templer et al (1998) record that their earliest observations indicated that the screen dominated the attention of most (although not all) students, and that some balance needs to be struck between directing students from paper to screen, and vice-versa. It was evident that some students are reluctant to move from screen to text, whereas the move the other way is more flexibly undertaken. It was noted that mathematical 'tools' are forged through use, in contrast to conventional tools that are first made and then used, and that this calls into question a sequence that seeks first to master a tool and then apply it. While in the paper this was in reference to the application of mathematics to chemistry, we may translate it for our present purpose, and ask as an open question as follows. Whether training in a manipulator such as *Mathematica, Derive, or Maple* requires prior time and effort, or whether a careful design can enable mathematics to be learned and applied contiguously with increasingly sophisticated manipulator use?

The program that is our focus of interest, is a mainstream course located between the extremes described respectively by Olsen (1998) and Shneiderman et al (1998). It represents a model that may be located comfortably within present university structures and resources. Issues associated with its implementation connect with those raised in Kent and Stevenson (1998), and Templer et al (1998). Like the latter we are concerned with the links between computer-controlled processes and their mathematical underpinnings, noting the similarities and differences between the respective symbolisms. With the former we share an interest in the range of questions raised by students as they work with the software, as well as their performance.

Attitudes to Mathematics and Technology

While there have been enthusiastic claims for the positive impact of technology on the teaching and learning of mathematics, systematic evaluations of impact have been harder to access. And while the study of *attitudes* in mathematics learning has a substantial history, the relationship between *attitude* and *performance* is not clear-cut although positive correlations have often been noted between these characteristics. Early claims that affective variables can predict achievement (e.g. Fennema and Sherman, 1978) have been balanced by later comments (e.g. Schoenfeld, 1989) indicating that research does not give a clear picture of the direction of causal relationships. The Tartre and Fennema (1995) comment that described *confidence* as the affective variable most consistently related to mathematics achievement is probably a safe summary of the position.

More recent studies have continued to pose the direction of the relationship between *attitude* and *performance* as an open question. Thus while Tall and Razali (1993) argued that the best way to foster positive attitudes is to provide success, Hensel and Stephens (1997) concluded that "it is still not totally clear whether achievement influences attitude, or attitude influences achievement", while Shaw and Shaw (1997) noted that among engineering undergraduates the top performing students (at entry) had a much more positive attitude to mathematics, and lower performing students a commensurately negative one – again leaving the direction of causality open.

The study of attitudes towards information technology (most frequently computers) has a shorter but more intensive history, probably because information technology, while newer, is all pervasive in its permeation of curriculum areas. In considering attitudes to information technology among tertiary students it is useful to note the disciplinary focus of the target groups in existing reports. While several studies have used Education students, Psychology students, and Social Work students, reports involving mathematics students appear harder to come by, although several have included affective variables when evaluating outcomes (see below). It is this very breadth of discipline background which has served to keep the investigation of attitudes to information technology at a general level, appropriate to the majority who will not be called upon to use computers in the same technical sense as mathematics students working intensively with specialized software.

The relevance of studying attitudes to information technology in conjunction with those relating to mathematics is emphasized and re-inforced by the increasing use of technological devices in mathematics instruction. Several studies refer incidentally to attitudinal impacts as well as proficiency measures and Mackie (1992) in an evaluation of computer-assisted learning in a tertiary mathematics course indicated six positive learning outcomes, three of which were related to attitudinal factors. Park (1993) in comparing a Calculus course (utilizing *Mathematica*) with a conventionally taught program, found some improvement in disposition towards mathematics and the computer in the experimental group. However Melin-Conjeros (1992), in comparing the performance of a group of Calculus students (equipped with limited access to *Derive*) with a control group, noted that the attitude of both groups decreased slightly. It is not generally clear in the mathematically focused studies just which 'attitudes' have been affected by technology, as the reporting tends to be non-specific. By inference it appears that it is 'attitude' to mathematics that is referred to, and we are led to consider the implications of technology in impacting upon component attributes. The consistent and strong relationship between mathematics confidence and performance noted previously (whatever the direction of causality), means that the implications of a nexus between technology and mathematics needs specific research attention. The broad reporting of studies on the use of technology in mathematics instruction makes it difficult to disentangle whether reported affective outcomes are associated with changed attitudes to mathematics, or are linked directly to the technology. So theoretically we are moved to ask about the interpretation of outcomes if students possess high mathematics confidence and motivation, but low computer confidence and motivation, and vice versa.

The Study Context

The studies presented in this paper address three purposes derived from questions that have been distilled from the wider literature, and from reports such as those referenced above.

1. *To design attitude scales for use in programs in which computer technology is specifically directed towards assisting mathematics learning.*
2. *To classify the range of student-generated questions that emerge when learning of mathematical content interacts with a symbolic manipulator environment.*
3. *To identify structural properties associated with the Maple environment that can be identified as linking task demand and student success.*

The research was conducted within first-year undergraduate mathematics courses taken by students studying mainly within Science and Engineering degree programs. As taught in 1999 and 2000 the courses comprised a lecture series complemented by weekly workshops, in which approximately 40 students are timetabled into a laboratory containing networked computers equipped with *Maple* software. The lecture room is fitted with computer display facilities so *Maple* processing is an integral and continuing part of the lecture presentation. To support their workshop activity students are provided with a teaching manual (Pemberton, 1997), continually updated to contain explanations of all *Maple* commands used in the course, together with many illustrative examples. During laboratory workshops two tutors and frequently the lecturer also, are available to assist the students working on tasks structured through the provision of weekly worksheets. The students can consult with the lecturer during limited additional office hours, and unscheduled additional access to the laboratory is available for approximately 5 hours per week. The course is also available on the Web a medium that is attracting increasing custom. Solutions to the weekly worksheets are provided subsequently.

The formal course assessment is constrained by departmental protocol and the availability of facilities. It comprised pen and paper exams at mid-semester and at end of semester (combined 80%), supplemented with three *Maple* based assignments (total 10%), and a mark assigned on the basis of tutorial work (10%). Consequently to succeed students must transfer their learning and expertise substantially from a software supported environment to written format. Thus they must be able to develop understanding through the symbolic manipulator medium with which they work, while simultaneously achieving independence from it. This involves the ability to learn, practice, consolidate and maintain pen and paper procedures that a *Maple* environment provides access to, and support for, but does not enforce. The educational implications of this characteristic need pursuing in their own right, but additionally attention is focused on the relationship between the mathematical demands of tasks, and their representation in a *Maple* learningscape. Although not a focus in the present paper it is noted, unsurprisingly, that a significant correlation (0.38) obtained between marks awarded for the *Maple* assignments and the end of year written test score.

Data Sources

In addition to performance data on the assessment measures, further student biographical data were obtained, and two additional instruments were administered in the first week of the course. These were firstly a mathematics test comprising secondary school content, deemed to be representative of prerequisite knowledge important for the course. Secondly, pertaining to research issue 1, a set of attitude scales (see below) to access the disposition of the students towards mathematics, computers, and their interaction in a learning setting. Only the latter is of interest in this paper.

The data for addressing issues 2 and 3 were obtained from two sources. For question 2 tutors assigned to the *Maple* workshops were provided with diaries in which they entered, on a weekly basis, examples indicative of the range, type, and frequency of questions raised by students in the course of their workshop activity. For question 3 a diagnostic test was scheduled 7 weeks after the course started. This test was a voluntary exercise, and comprised a series of questions to be addressed with the assistance of *Maple* in its laboratory context. Its purpose was to provide formative feedback to the students on their performance, and ranged from simple school level manipulations to new material introduced in the tertiary program. Three sample questions are included in the appendix, together with their *Maple* solutions. As an incentive the test was directly relevant to preparing for the formal assessment at the end of semester, for the procedures required are ones that the students need to be proficient with, irrespective of software support. Additionally several questions contained an explanatory component, where the students were required to interpret the meaning of graphical output. Two sets of data were obtained from the tests, which were analyzed and marked by two of the course tutors using criteria designed by the researchers. One of these involved the recording of correct and incorrect solutions, except that for this purpose the quality or indeed presence of a final interpretation of graphical output was not taken into account. This meant that the correct/incorrect dichotomy was on the basis of *Maple* operations only. The second set of data was obtained from an analysis of errors that led to incomplete or incorrect answers.

Two further pieces of information are relevant to a discussion of data collection. With respect to attitude measures we note that while mathematics has existed in an established form for a substantial time, the computing environment is changing and with it the degree of associated experience of beginning undergraduates. It is therefore pertinent to consider whether observed structural differences between mathematics and computer based affective responses on attributes such as *confidence* and *motivation* will diminish with time, or whether they represent distinctive sets of characteristics with a permanent presence. To this end, in addition to the data obtained from year 2000 students (N~160), we include parallel data obtained from administering the same instruments to a 1997 cohort (N~140) in a corresponding course. Secondly, while the data that addresses questions 1 and 2 come from the year 2000 cohort, that which is used to address question 3 is drawn from the 1999 enrolment (N~250). This was not originally intended and, in a rather bizarre circumstance, is attributable to the Sydney Olympics. University timetables were adjusted to fit in with the games, and a last minute re-scheduling of the formal mid-semester test effectively destroyed the administration of the diagnostic *Maple* based test, which had been arranged for the same week.

Instrument Design

Attitude scales (question 1)

Pilot work on such scales (Galbraith and Haines, 1998) was conducted with undergraduate students in introductory mathematics subjects at City University (London).

Given the purpose of developing scales for use in settings involving interaction between technology and mathematics learning, we found the positions articulated by Hart (1989), Mandler (1989), and McLeod (1989, 1992) to be helpful in fashioning the approach to our definition of terms and hence instrumentation. We shall refer to this as the HMM classification in which the ordering *beliefs*, *attitudes*, *emotions* represents increasing affective involvement, decreasing cognitive involvement, and decreasing stability. Beliefs are viewed as mainly cognitive in nature being built up slowly over time, while attitude may be viewed as the end result of emotional reactions that have been internalized and automatized (McLeod, 1989) to generate feelings of moderate intensity and reasonable stability. With the decay of the emotional content with time, the response becomes more stable, and hence amenable to access through questionnaires and interviews. Emotions, as hot reactions, cannot reasonably be measured by such dispassionate means as questionnaires, and their role has been relatively underplayed in terms of research. We have adopted the HMM classification as the basis from which to develop our instrumentation. The distinction between an *attitude* and a *belief* is tenuous to a degree – we have endeavored to seek an *attitude* focus by wording items so that the respondent is personally involved:

E.g. I feel more confident of my answers with a computer to help me; rather than
Computers help people to be more confident in obtaining answers

The students for whom our measures are designed are tertiary undergraduates in mathematics courses; having made this career choice-whereby mathematics has been selected as both useful in pursuing career aspirations, and as a subject compatible with themselves as individuals. Hence while we retain an overall monitoring interest, the categories of gender and usefulness, that have figured prominently in other attitude studies, (E.g. Fennema and Sherman, 1976), did not play a dominant role in our design. Two of the nine attributes (confidence and motivation) represented in the Fennema-Sherman formulation have been reflected in scale development, with appropriate items constructed for use by undergraduates. The choice of these attributes was influenced strongly by the total purpose of designing instruments for use when computer technology is used in the teaching/learning context. We have chosen *confidence* and *motivation* because of their extensive appearance in the literature for both mathematics and technology, and because of their potential for discriminating between attitudes when technology and mathematics interact. These four scales are designed to measure attitudes on both dimensions so that such differences can be identified and their implications noted. In particular the choice of *confidence* and *motivation* enables two circumstances of particular interest to be identified viz situations where students hold strong positive feelings towards mathematics and negative feelings towards technology, and vice-versa. And of course *confidence* and *motivation* are two constructs that have been strongly and consistently linked with mathematics achievement over many years as discussed previously.

A further scale was deemed desirable, to include factors important for the learning context, that are not accessed by the separate computer and mathematics scales. This scale provides a measure the degree of interaction between mathematics and computers that students perceive they apply in learning situations, noting that the interactive significance of the learning and instructional context has been emphasized (E.g. McLeod 1989). In a computer environment students may simply respond to the screen or be active in note making, summarizing, and experimenting. Indeed they may choose not to utilize technology when it is available and relevant (Boers and Jones, 1994). The physical separation of the learning components; pen and paper, computer screen, and human brain adds a further dimension to the co-ordinating processes required for effective learning strategies. The *computer-mathematics interaction* scale assesses the extent to which students bring their mathematical thinking into active inter-play with the computer medium. In fact a sixth scale (*mathematics engagement*) was initially a part of the attitude study. However this scale correlated strongly with *mathematics motivation* and it was decided nothing was added by retaining it in the analysis. It was therefore omitted from the year 2000 data collection.

Within each scale the items were arranged randomly with half requiring the reversal of polarity at the coding stage. Students were asked for a measure of their agreement (or rejection) with respect to item wording, which resulted in a 13 point Likert scale. The item groups were presented in such a way that the underlying constructs were unknown to the students.

The questionnaire items were presented on a Likert scale in which students were asked to express their agreement or disagreement with a statement to describe their own viewpoint. The choice of an eight-item scale represents a trade-off between higher reliability measures that can be obtained using more items, and the practical need for an instrument that does not induce respondent fatigue. The scale items were theoretically determined from the respective underlying constructs and from cognate literature, and the reliability of the scales assessed by means of α coefficients. We did not proceed down the common route of using factor analysis on a smorgasbord of possible items. A discussion of validity and reliability will be included with a discussion of the data in the 'outcomes' section.

Student-generated questions (question 2)

The questions asked by students during laboratory sessions were systematically recorded in tutor diaries. The diary categories were selected to cover the two principal areas of interest-questions associated with general mathematical concerns, and questions stimulated by the technical use of *Maple*. Additionally questions prompted by procedural needs were noted. A pilot study in 1999 supported an expansion and refinement of question categories to those represented Table 5. During each laboratory session the tutors, using a checklist, noted the frequency with which different question types were asked. They also recorded a typical example of each type that arose in a given session.

Impact of Maple Environment on performance (question 3)

Error patterns:

The error analysis from the *Maple* based test results generated a range of individual flaws (over 600 in total), which could be coarsely grouped into four main categories as shown in Table 6 in the 'outcomes' section. Again these are judgment based with an element of subjectivity-they are essentially errors of commission. Errors of omission, as evidenced for example by failure to invoke appropriate commands, could not be so readily quantified. As far as operations with *Maple* are concerned it seemed we could identity two categories (at least) of facility. One of these is the requirement of accurate syntactical representation of common elementary operations, such as are represented in the first row in Table 6. The other is the more sophisticated and demanding selection and specification of functions to achieve identified mathematical ends. Clearly there is interaction between mathematical understanding and function specification, for if the former is flawed the wrong selection may be made or functions combined inappropriately. Alternatively if the mathematics is correct, the desired outcome can be defeated by mistakes in the technical detail of function specification.

Complexity:

We sought to relate performance to the influence of the two categories just discussed, which have been labelled SYNTAX and FUNCTION respectively.

SYNTAX: refers to the general *Maple* definitions necessary for the successful execution of commands. These include the correct use of brackets in general expressions, and common symbols representing a specific syntax different from that normally used in scripting mathematical statements (such as *, ^, Pi, g:=).

FUNCTION: refers to the selection and specification of particular functions appropriate to the task at hand. Specific internal syntax required in specifying a function is regarded as part of the FUNCTION component, including brackets when used for this purpose. Complexity is represented by a simple count of the individual components required in successful operation. We now illustrate how these definitions work, by applying them to the examples given in the appendix.

Q2. SYNTAX: Incidence of ^ [2] *plus* * [2]; total=4.

FUNCTION: General structural form of factor(argument); factor [1] *plus* ( ) [1] *plus* argument entry [1]; total=3.

Q8. SYNTAX: Incidence of ^ [1] *plus* *[2] *plus* ( ) [2] *plus* x1[1] *plus* := [1]; total=7.

FUNCTION: General structural form of plot(function, domain); plot [1] *plus* ( ) [1] *plus* , [1] *plus* function entry [1] *plus* domain entry [1] *plus* domain specification [1]; sub-total=6.

General structural form of fsolve (function, domain); sub-total [5] *plus* domain specification [1]; total =12.

Q14. SYNTAX: Incidence of* [2] *plus* ( ) [3]; *plus* y [1] *plus* := [1]; total=7.

FUNCTION: General structural form of plot(function, domain); sub-total [5] *plus* domain specification [1];

General structural form of int(y, integ interval); sub-total [5] *plus* (subtraction) [1] *plus* integration interval specifications [2]; total=14.

Similar pairs were assigned to each of the 14 questions forming the test sample. Our diagnostic approach involves scoring on a correct/incorrect basis, as we are not (in this analysis) concerned with apportioning partial credit as would be necessary if grading student performance. The success rate on the questions is given by the fraction of students (N~ 250) obtaining the correct answer. We can regard these as providing a measure of the probability of success of a student from this group on the respective questions. For the questions in the Appendix the respective values are 0.89, 0.26, and 0.14

Outcomes

Attitude data (question 1)

To illustrate sample items we include below items whose responses contributed most strongly to the scale score; polarities have been adjusted so that a higher score means more of the property described by the scale label. We include for two of the scales the positively worded item(s) attracting the strongest support, and the negatively worded item(s) invoking the strongest rejection. (A = year 2000, B = year 1997). A1&B2 etc means that the item was the strongest choice of 2000 students, and second strongest choice of 1997 students.

mathematics confidence: I can get good results in mathematics (A2&B1)
*No matter how much I study, math is always difficult for me (A1&B1)

computer confidence: I am confident I can master any computer procedure that is needed for my course (A1&B1)
*As a male/female (*cross out that which does not apply*) I feel disadvantaged in having to use computers (A1&B1)

* Negatively worded item involving scale reversal

Scale means are provided in Table 1 with 1997 data in brackets.

Table 1: Scale Means

mathematics confidence	8.6 (7.8)	computer confidence	8.7 (7.4)
mathematics motivation	8.0 (7.2)	computer motivation	7.7 (6.7)
		comp/math interaction	7.3 (6.4)

The study did not set out to compare the *level* of student response between the 1997 and 2000 groups, for interest is focused on the structural relationships between the mathematics and computer responses. It is observed however that the relative magnitude of means has a similar ordering within each group.

Scale reliabilities

These were obtained for each scale as follows-1997 data in brackets (see Table 2).

Table 2: Scale Reliabilities (Cronbach α)

mathematics confidence	0.81 (0.85)	computer confidence	0.85 (0.88)
mathematics motivation	0.82 (0.84)	computer motivation	0.81 (0.86)
		comp/math interaction	0.71 (0.70)

The scales are coherent with reliabilities from strong to moderate, and with all items contributing. The bringing together of disparate properties to address interaction issues, has unsurprisingly resulted in a somewhat lower α value for that scale than for closely defined concepts like confidence and motivation.

Scale validity

This rests primarily upon the theoretical base behind the construction of the scales. Additional structural evidence may be inferred from the sample items given above. For example the two items attracting the strongest responses for *mathematics confidence* (expecting good results, and rejecting that mathematics is difficult irrespective of effort), are both centrally to do with confidence. The coherence of the scale as indicated in the α value then supports the argument for validity without examining each additional item. Similar arguments apply to the other scales.

Differences in Attitudes to Mathematics and Computing

A main purpose in this research was to investigate the extent to which attitudes to computer use and to mathematics represent different inputs into technology based teaching contexts involving mathematics learning

Table 3: Inter-scale correlations

	mconf	mmotiv	cconf	cmotiv	cmint
mconf		.51(.68)	.22(.21)	-.04(.19)	.04(.16)
mmotiv			-.07(.23)	.00(.29)	.15(.26)
cconf				.62(.75)	.56(.58)
cmotiv					.65(.66)

Consider first, correlations between the five scales (Table 3; 1997 data in brackets). The entries in Table 3 indicate that the confidence and motivation scales are strongly associated within mathematics (0.51,0.68) and within computing (0.62,0.75) but they are less strongly associated across the areas. This is shown by the weak correlation, for example, between mathematics confidence and computer confidence (0.22,0.21). The computer-mathematics interaction scale is more strongly associated with the computer confidence (0.56,0.58) and computer motivation (0.65,0.66) scales than with the mathematical scales where correlations are weak. This suggests that computer attitudes are more influential than mathematical attitudes in facilitating the active engagement of computer related activities in mathematical learning. These results suggest that a Factor Analysis using the five scales as input variables with a two-factor solution as goal is appropriate. Using oblimin rotation (SPSS) following a principal components analysis the loadings shown in Table 4 were obtained. The two-factor solution confirms that the computer and mathematics related scales define different dimensions with computer properties dominant in the interaction scale. (Year 1997 data again in brackets.)

Table 4: *Factor Pattern Matrix*

	Factor 1	Factor 2
mconf	.02(-.06)	**.88(.87)**
mmotiv	-.02(.03)	**.87(.89)**
cconf	**.84(.89)**	.05(-.03)
cmotiv	**.89(.90)**	-.11(.02)
cmint	**.85(.83)**	.06(.02)

Percentage of variance 75.3(69.7)

Student-generated questions (question 2)

A total of over 1300 questions indicative of the range of concerns displayed by students when working mathematically in a *Maple* environment was assembled from the tutor diaries. The categories were selected using a mix of empirical judgment, theoretical positioning, and the results of a pilot study in the previous year. The distribution is shown in Table 5.The number of questions per category varied from a maximum of 333 (24.6%) to a minimum of 29 (2.2%). The number of questions in which some aspect of *Maple* was unequivocally involved exceeded 80%.

Table 5: *Student Question Types*

Question Category	Percentage
1. Identify problem caused by a typo (TYPO)	8.4%
2. Resolve syntax error (SYN)	24.6%
3. Problem with function choice (FCHCE)	4.2%
4. Problem specifying function (FSPEC)	14.6 %
5. Stuck on mathematics (STMATH)	14.9 %
6. Procedurally stuck on Maple (STMAPLE)	19.5 %
7. Interpreting aspects of output (INTOUT)	11.6 %
8. General procedural (PROC)	2.2 %

Impact of Maple Environment on performance (question 3)

The error analysis from the *Maple* test results generated a range of individual flaws (over 600 in total), which could be coarsely grouped into four main categories as shown in Table 6. Again these are judgment based with an element of subjectivity-they are essentially errors of commission. Errors of omission, as evidenced for example by failure to invoke appropriate commands, could not be so readily quantified. Rows 2 and 3 (Table 6) considered jointly, confirm the respective significance of function specification and mathematical activity, an error distribution that is also consistent with the major categories of questions asked by students in a laboratory setting (see Table 5 above).

Table 6: *Test Error Classification.*

Error Category	Percentage
1. Syntax related errors (*,( ), ^ etc.)	31.1 %
2. Function choice and specification errors	17.3%
3. Errors in mathematics	17.3 %
4. Comment omitted or inadequate	34.3 %

Regression Analysis

Based on a review of the 250 (approx) scripts received, it was determined that 16 questions had been attempted by the whole student group. For technical reasons two of these were deemed unsuitable for inclusion, so that responses to 14 questions formed the final data set. A linear regression analysis was performed using 'fraction successful' as a measure of the dependent variable (probability of success), and SYNTAX and FUNCTION as input variables (Tables 7-9). In the tables, significance at the .05, .01, and .001 levels are designated by *, **, and *** respectively.

Table 7: Regression

Regression statistics	
Multiple R	0.8710
R Square	0.7586
Adjusted R Square	0.7148
Standard Error	0.1419
Observations	14

Table 8: Analysis of Variance

	Df	SS	MS	F	Sig F
Regression	2	0.6957	0.3479	17.29	.0004***
Residual	11	0.2213	0.0201		
Total	13	0.9171			

Table 9: Other Statistics

	Coefficients	Standard Error	t Stat	P-value
Intercept	1.0947	0.0961	11.383	2E-07
SYNTAX	-0.0482	0.0168	-2.874	0.015*
FUNCTION	-0.0396	0.0122	-3.246	0.008**

Thus both the SYNTAX and FUNCTION complexity measures contributed significantly to the task demand of the questions.

Reflections

Pedagogies to support the increasing use of technologies in undergraduate teaching are still in the process of development or refinement, and within this enterprise the interaction between mathematics and technology is of significant importance. With respect to question 1 we note firstly the properties independently confirmed among students from different cohorts at different times. Secondly the strong correlations between *confidence* and *motivation* within mathematics and computing respectively, suggest that one such scale for each might suffice if a concise instrument is envisaged. *Computer/mathematics interaction* should be retained as it represents an important indicator of student involvement. Two further potentially significant inferences emerge, given the robust behavior of the scales across time and place. Firstly the confirmation that attitudes to mathematics and computing occupy different dimensions (the respective factors are almost orthogonal), with interaction loading with the computer scales. Secondly, at least an interim answer to the following question. Given that students' prior access to and experience with computers is continually increasing, will differences identified between mathematics and computer based affective responses to parallel attributes such as confidence and motivation diminish with time, or do they represent distinctive sets of characteristics with a permanent presence in computer-assisted mathematics learning. The data so far suggest the latter!

With respect to our investigations of questions 2 and 3 we can relate to the comment of Ramsden (1997) that "the impact upon educational practice of powerful software…has been less profound than optimists hoped or pessimists feared". Almost all reports contain statements tempering enthusiasm with caution, or disappointment with optimism. A continuing challenge is articulated by Olsen (1999) following a description of the most extensive budget driven, automated, attempt at mass produced learning that we have so far identified.

> Instructional software issues are unlikely to be resolved quickly... If we want the software to help at all… it's got to understand how students might misconceive what is presented to them--and to figure that out from the student's response. And right now, only people do that well. (p. 35)

Regarding question 2 the patterns evident in Table 5 reinforce that when students interact with mathematics through technology questions are generated rapidly and their scope is vastly increased. We can identify at least four types of inquiry from the responses: Those that are simply procedural (what to do next); those that are mathematical in the traditional sense; those that are software related (syntax and symbols); and those that are generated by the interaction of mathematics with software (function choice and specification). The intensity and scope of student questioning has ballooned in comparison with traditional practice classes, with software the major contributor through properties of fast processing, scope for formatting and specification errors, and just plain knowledge blocks in bringing the mathematics and software together, together with student initiative in exploring. In examining the analysis relevant to our third question, we observe that while achieving more rapid and efficient closure to algorithmic procedures the use of *Maple* has not reduced the need for the mathematical attributes of understanding and attention to detail. We note this in the significant impact of the variables SYNTAX and FUNCTION on success rate. SYNTAX errors penalize those who lack sufficient care in expressing their work symbolically, while the demands imposed by FUNCTION are proportional to the principles and sophistication of the associated mathematics. On the other hand, for those students who possess conceptual understanding and due regard for precision, the *Maple* environment has provided a means to progress rapidly and successfully at a greater rate than could otherwise be achieved. Our conclusion to this point is that there is no 'free lunch' (indeed laboratory tutors are lucky to get lunch at all). The propensity of students to alter their approach to reduce the learning potential available to them (Templer, Klug, and Gould, 1998) is apparent. It is hoped that as student performance is mapped more carefully, and lessons learned from their responses to both mathematical tasks and in teaching situations, new insights for teaching-learning options will be identified. New properties that emerge from the mutual interaction of students, mathematics, and technology can support new approaches extending beyond the models that thus far appear to have motivated many of the proponents of automated learning; goals of doing faster and more cheaply that which was done formerly with blackboard, chalk, and paper (Olsen, 1999; Thorpe, 1998). These are limited goals indeed. The present research contributes to this broader endeavor, both in terms of identifying and classifying student responses to laboratory activities, and in linking mathematical demand to the complexity of manipulator operations and task success.

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Appendix

Sample Questions

(Questions in italics: *Maple* commands in bold: *Maple* output in ordinary type)

Q2. Factorize $x^3 - 6x^2 + 11x - 6$

Maple Solution

➢ **factor(x^3-6*x^2+11*x-6);**

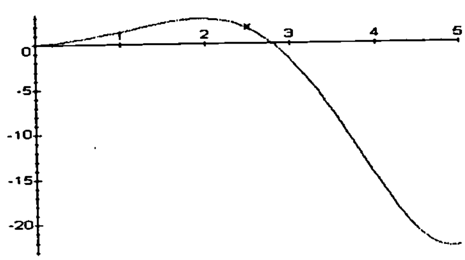
(x - 1) (x - 2) (x - 3)

Q8. Find where the graph of $x^2 \sin x + x \cos x$ *for* $0 \le x \le 5$ *is:*

(a) above the $x-axis$ *(b) below the* $x-axis$ *(c) cuts the* $x-axis$.

Maple Solution

> **plot(x^2*sin(x)+x*cos(x),x=0..5);**



> **x1:=fsolve(x^2*sin(x)+x*cos(x),x=2..3);**

x1 :=2.798386046

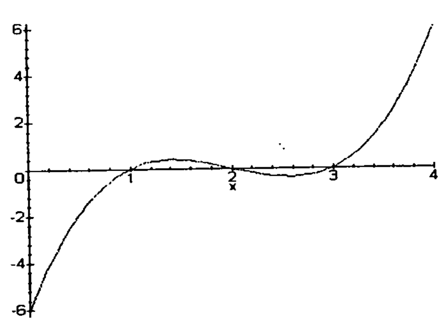
****

Q14. Plot the graph of f(x) = (x-1)(x-2)(x-3) and use this to find the physical area under the graph from x= 1 to x= 3.

Maple Solution

> **y:=(x-1)*(x-2)*(x-3);**

➢ **plot(y,x=0..4);**



➢ **int(y,x=1..2)-int(y,x=2..3);**

➢ 1/2

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